Some PDE-ODE approaches for the non-stationary dynamics of an asexual population facing arbitrary environmental change

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Résumé

Many populations experience conditions that change in time over short timescales, where demographic and evolutionary dynamics cannot be separated. The typical theoretical approach to model changing environments (in space and/or time) is to consider adaptation at a set of traits, for an optimum which position depends on the environmental conditions. This formalism, although simple in essence, implies several key properties that make the demographic and evolutionary dynamics difficult to model explicitly: pervasive epistasis, infinitely many genotypes, time varying selection etc. Several decades of theoretical developments have tackled this general model by coupling the 'fitness landscape' with particular environmental scenarios: fixed, steady change, oscillating or stationary stochastic optimum movement. Most analytical insight was gained by focusing on weak selection acting on freely recombining genomes (infinitesimal model and quantitative genetics approach), and by only characterizing the long-term stationary regimes of adaptation (or maladaptation), mostly under deterministic approximations.
However many key situations (e.g. relevant to treatment outcomes or invasions) rather correspond to asexuals in non-stationary regimes. Here, we study the evolutionary (and sometimes demographic) dynamics of a large asexual population, at or away from stationarity, when the optimum moves arbitrarily along a fixed direction in phenotype space (which includes most classic scenarios alluded to above). Analytical progress is made by considering a PDE for a characteristic function of the fitness distribution, that is coupled with a demographic ODE when considering non-stable population sizes. In the simplest case of adaptation to a fixed optimum (abrupt change) and fixed population size, the PDE (nonlinear and nonlocal) can be obtained explicitly as a limit of the more exact stochastic problem by using diffusion generators or a Feynman-Kac formula. In all cases, the PDE-ODE system can be closed and studied numerically (although it proves difficult in the moving optimum context). Finally, a weak selection strong mutation regime can be considered where fully analytic solutions over time can be found in the general case.
Beyond providing deterministic insight, the approach can be coupled with stochastic demographic models (diffusion approximation) to compute the probability of extinction vs. establishment in the face of arbitrary environmental change, in the weak selection strong mutation regime. Some extensions to other ecological or evolutionary contexts will be discussed if time allows.

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